A PRODUCTION-FUZZY INVENTORY MODEL WITH DETERIORATION

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Abstract

This study explores the effect of learning on fuzzy considering the fuzzy demand in the EOQ model for deteriorating materials. The crisp equivalent form of the fuzzy objective function is obtained by using the graded mean method. The number of fillers that optimizes the fuzzy objective function is obtained. The model is extended by using learning in a fuzzy and an algorithm has been developed to determine the minimize the total cost. Numerical charts are provided for the sample under a crisp, and fuzzy environment.

Keywords: *EOQ Inventory model, graded mean method, Trapezoidal Numbers*

1.Introduction

The deterioration of an object is considered to be the loss of a part or all of its value over time. Materials such as electronics and style materials suddenly become obsolete, detergents, chemicals, prescription drugs and blood have a fixed shelf life, but their usefulness does not diminish in shelf life. Radioactive materials are subject to high-rate decay. Items like new products lose their usefulness in their lifetime. It is very difficult for all manufacturing industries to store highly volatile substances such as liquids and blood, which can spoil over time. Therefore, it is important to discuss the behaviour of these types of articles.

In this direction, Carey and Schrader [1] were the first authors to consider the effect of article spoilage on the inventory model. They discussed the general EOQ (economic order scale) model with direct deterioration and exponential deterioration. Goyal and Giri [2] provide a comprehensive overview of the deterioration of inventory literature. Manna et al. [3] Provided EOQ model for non-degradable materials with variable demand over time and partial delay. Montal et al. [4] Inspected inventory models for defective products, including marketing results with varying production costs. Goyal [5,6], on the other hand, followed the policy of economic discipline of articles that deteriorated indefinitely.

Maihami and Karimi [7] improved the price and top-up policy for non-immediate goods with random demand and incentive measures. Maihami & Kamalapadi [8] is considered a freight control model for goods that do not have instant decay with partial delay and demand depending on time and price. Dye [9] examined the effect of investing in security technology in an inventory model without immediate decay. Gorishit al. [10] Developed an optimal pricing and ordering policy for goods that are not immediately deterioration under inflation and customer revenue.

In 1965, Jadeh [11] put forward a remarkable concept of ambiguous compilation theory, which has been used successfully and consistently in various fields of science and technology. Another researcher according to the work of Chang and Jade [12] and Chen et al. [13,14] Large-scale analysis to enrich fuzzy numbers around the world and to make assumptions of uncertainty theory, which eventually led to many interesting conclusions in this area. Then, in the further course of the investigation, the notion of ambiguity is extended to sets of fuzzy intervals.

Fuzzy numbers that play a useful and important role in the problem of mathematical modelling and statistical calculation. Fuzzy numbers can be viewed from two different perspectives: their membership function or their-cuts. The two ways of looking at fuzzy numbers are similar, and depending on the details we want to explore, one may be better than the other. Of all the types of fuzzy numbers, triangular and trapezoidal numbers are the most commonly used, and their names are derived from the form derived by representing their membership function in the Cartesian plane. However, if we look at the fuzzy number in terms of its α-cuts, we get a really spaced view of the fuzzy number

Also, trapezoidal number is used, After that, we consider the inventory management model in fuzzy context, where we used the proposed defuzzification methods. Finally we modified the sensitivity analysis Parameters of the model. Finally, we noticed because of that fuzzy existence, it will give us good result rather than the crisp one.The article is structured as follows: preliminaries of the proposed work is given in Section 2 , Provides notation and assumptions. Model formulation is discussed in Section 3. Numerical examples, comparisons between models and sensitivity analysis are presented to illustrate the model in Section 4. Finally, the conclusion of the model have been made in section 5.

2.Methodology

2.1 Fuzzy Set

A fuzzy set $\tilde{A}$ on the given universal set X is a set of ordered pairs $\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) : x \varepsilon X \}$ where $\mu_{\tilde{A}} : x \rightarrow [0,1]$ is called membership function or grade membership. The membership function is also a degree of compatibility or a degree of truth of x in $\tilde{A}$.

2.2 Generalized Fuzzy Number

Any fuzzy subset of the real line R, whose membership function satisfies the following conditions, is a generalised fuzzy number.

i. $\mu_{\tilde{A}}(x)$ is a continuous mapping from R to the closed interval [0, 1].
ii. $\mu_{\tilde{A}}(x) = 0$, $-\infty \leq x \leq a_1$
iii. $\mu_{\tilde{A}}(x) = L(x)$ is strictly increasing on $[a_1 , a_2]$
iv. $\mu_{\tilde{A}}(x) = 1$, $a_2 \leq x \leq a_3$
v. $\mu_{\tilde{A}}(x) = R(x)$ is strictly decreasing on $[a_3 , a_4]$
vi. $\mu_{\tilde{A}}(x) = 0$, $a_4 \leq x \leq \infty$ where a_1, a_2, a_3 and a_4 are real numbers.

2.3 Trapezoidal Fuzzy number

The fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$ where $a_1 < a_2 < a_3 < a_4$ are defined on R is called the trapezoidal fuzzy number , if the membership function $\tilde{A}$ is given by ,

$$\mu_{\tilde{A}}(x)=\begin{cases}0, \; if \quad x < a_1 \, or \, x > a_4 \\ \dfrac{x-a_1}{a_2-a_1}, \; if \; a_1 \le x \le a_2 \\ 1, \; if \quad a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4} if \quad a_3 \le x \le a_4 \end{cases}$$

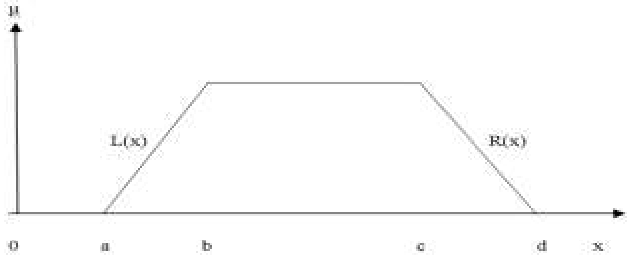


Fig.1: Trapezoidal Fuzzy Number

2.4.Graded mean integration representation method.

Chen and Hsieh [1999] introduced Graded mean Integration Representation Method based on the integralvalue of graded mean h-level of generalized fuzzy number for defuzzifying generalized fuzzy number. Here, we first define generalized fuzzy number as follows:

Suppose $\tilde{A}$ is a generalized fuzzy number as shown in Figure 2.

It is described as any fuzzy subset of the real line R, whose membership function

$\mu_{\tilde{A}}$ satisfies the following conditions.

1$\mu_{\tilde{A}}$ (x) is a continuous mapping from R to [0, 1],

2. $\mu_{\tilde{A}}$ (x) = 0, $-\infty < x \le a_1$

3. $\mu_{\tilde{A}}$ (x) = L(x) is strictly increasing on $[a_1, a_2]$,

4. $\mu_{\tilde{A}}$ (x) = W_A, $a_2 \le x \le a_3$,

5$\mu_{\tilde{A}}$ x) = R(x) is strictly decreasing on $[a_3, a_4]$,

6. $\mu_{\tilde{A}}$ (x) = 0, $a_4 \le x < \infty$,

where $0 < W_A \leq 1$ and a_1, a_2, a_3 and a_4 are real numbers. This type of generalized fuzzy numbers are denoted as $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)LR$. When $w_A = 1$, it can be formed as $\tilde{A} = (a_1, a_2, a_3, a_4)LR$. Second, by Graded Mean Integration Representation Method, L^{-1} and R^{-1} are the inverse functions of L and R respectively and the graded mean h-level value of generalized fuzzy number

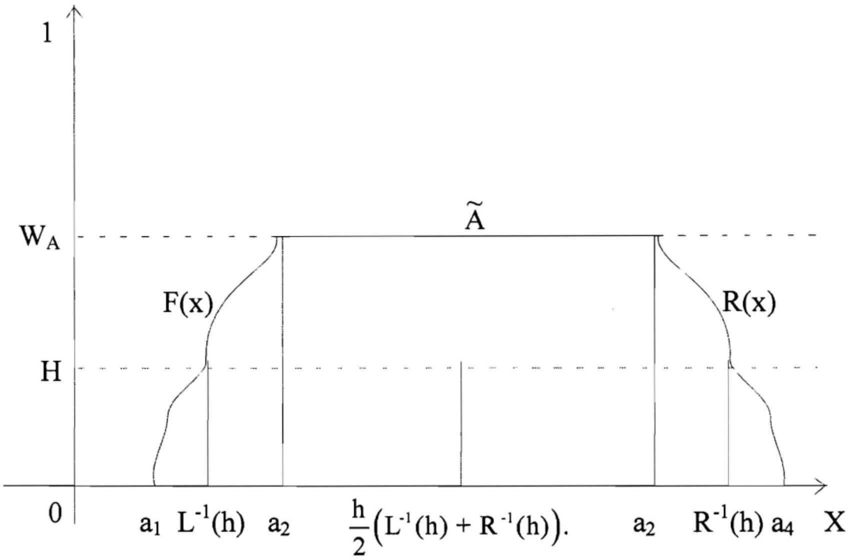


Figure 1.The graded mean h-level value of generalized fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4; w_A)LR$

$\tilde{A} = (a_1, a_2, a_3, a_4; w_A)LR$. is given by $\frac{h}{2}(L^{-1}(h) + R^{-1}(h))$ (see Figure 2). Then the graded Mean Integration Representation of $P(\tilde{A})$ with grade w_A, where

$$P(\tilde{A}) = \frac{\int_0^{wA} \frac{h}{2}(L^{-1}(h) + R^{-1}(h))dh}{\int_0^{wa} h dh}$$

$$= \frac{a_1 + 2a_2 + 2a_3 + a_4}{6} \qquad (2.4.1)$$

3.Mathematical Model

3.1 Notations and Assumption

The following notation and assumptions are thought of to develop the model:

Notation:

To develop the model we tend to use the virtually similar notation as on Yan et al. [15].

P production rate

C setup cost for a production batch

H_s holding cost for the supplier

O ordering cost for the buyer

D constant demand

H_b holding cost for the buyer

Db area under the buyer's inventory level

F constant transportation cost per delivery

P_d deterioration rate

C_b deterioration cost for the buyer

C_s deterioration cost for the supplier

n numbers of deliveries per production batch,

Q production lot size per batch cycle

V the unit variable cost for order handling and receiving

ITC the total cost of the system

Assumptions:

1. The production inventory system only produces one type of item.
2. The demand is considered constant and deterministic.
3. The buyer's inventory and demand information is provided to the supplier.
4. Assume the supplier's production rate is constant and P> D.
5. Amounts and discounts for immediate payment are not taken into account.
6. The buyer bears the shipping and other handling charges.

3.2. Proposed Inventory Model in Crisp Sense

In this model, the integrated total cost consists of buyer and vendor ordering cost, inventory holding cost, Deterioration cost , Setup cost, Transportation cost and handling cost.

The joint integrated total cost per unit time derived in Yan et al. [15] is the sum of the following elements, The different types of cost are as follows:

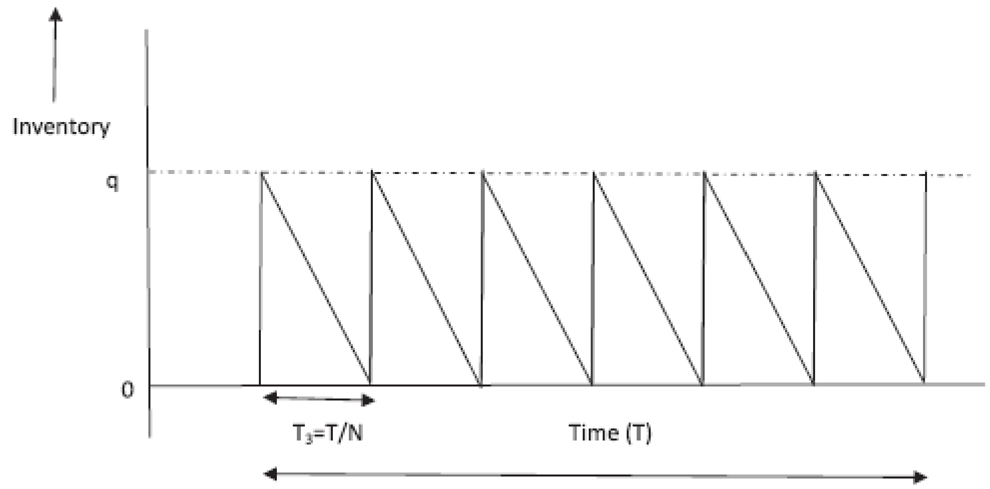


Fig. 1. Inventory model for the buyer's: inventory versus time

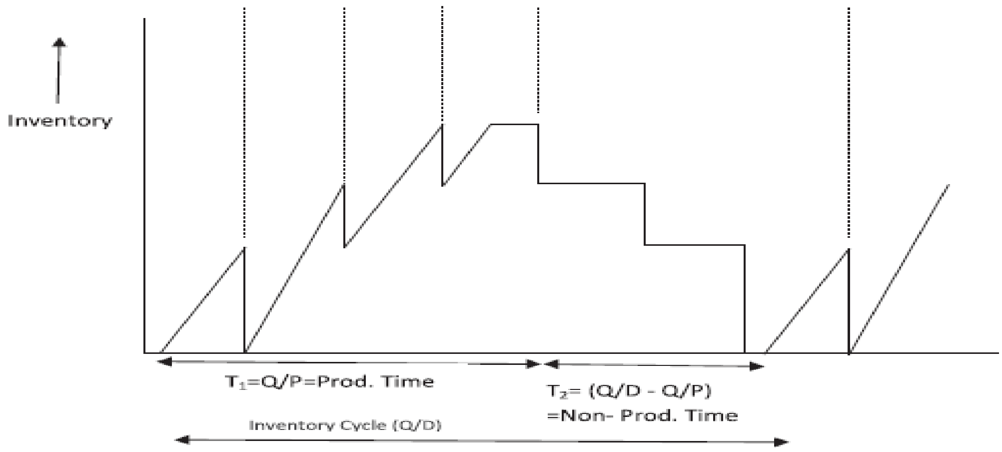


Fig 2.Inventory model for the supplier's: inventory versus time

1. Ordering cost per unit time = O $\left(\frac{D}{nQ}+\frac{D}{2n}\right)$.

2. Buyer Holding cost per unit time = $H_b\ \frac{Q}{2}$.

3. Supplier's Holding cost per unit time = $H_s\ \frac{Q}{2}\left(\frac{(2-n)D}{P}+n-1\right)$

4. Buyer Deterioration cost per unit time = $C_b\ P_d\ \frac{Q}{2}$.

5. Supplier's Deterioration cost per unit time $= C_s P_d \frac{Q}{2}\left(\frac{(2-n)D}{P}+n-1\right)$

6. Transportation cost and handling cost per unit time = (nF+VnQ) $\left(\frac{D}{nQ}+\frac{D}{2n}\right)$

7. Setup cost per unit time $= C\left(\frac{D}{nQ}+\frac{D}{2n}\right)$.

The average total cost of the production inventory model for the entire system is

ITC(Q,n) =

$$\left\{\left(\frac{D}{nQ}+\frac{P_d}{2n}\right)\left(O+C+nF+VnQ\right)+\frac{Q}{2}\left(H_b+C_bP_d\right)+\left[\left(\frac{Q}{2}\left(\frac{(2-n)D}{P}+n-1\right)\right)\left(H_s+C_sP_d\right)\right]\right\}$$

$$ITC(Q,n) = \begin{cases} \frac{D(O+C+nF)}{nQ}+DV+\frac{P_d}{2n}(O+C+nF) \\ +\frac{Q}{2}\left\{\left(H_b+C_bP_d+VP_d\right)+\left[\left(\frac{(2-n)D}{P}+n-1\right)\left(H_s+C_sP_d\right)\right]\right\} \end{cases} \text{----(1)}$$

Theorem : For fixed n ITC(Q, n) is convex in Q.

Proof :

Taking the first and second partial derivatives of ITC(Q, n) with respect to Q, we have

$$\frac{\partial ITC(Q,n)}{\partial Q} = -\frac{D(O+C+nF)}{nQ^2}+\frac{\left\{\left(H_b+C_bP_d+VP_d\right)+\left[\left(\frac{(2-n)D}{P}+n-1\right)\left(H_s+C_sP_d\right)\right]\right\}}{2} \text{--(2)}$$

Hence, for fixed n ITC(Q, n) is convex in Q, since

$$\frac{\partial^2 ITC(Q,n)}{\partial Q^2} = \frac{D(O+C+nF)}{nQ^3} > 0$$

we obtain optimal order quantity Q by setting Eq. (2)to zero as

$$\frac{\partial ITC(Q,n)}{\partial Q} = 0$$

$$Q=\sqrt{\frac{2D(O+C+nF)}{n\left\{(H_b+C_bP_d+VP_d)+\left[\left(\frac{(2-n)D}{P}+n-1\right)(H_s+C_sP_d)\right]\right\}}}\text{ --------------(3)}$$

3.3. Proposed Inventory Model in Fuzzy Sense

We consider the model in fuzzy environment. Since the Demand is fuzzy in nature, we represent them by trapezoidal fuzzy numbers. Let D : fuzzy Demand per unit time . Now we fuzzify total cost given in (1), the fuzzy total cost is given by:

$$I\tilde{T}C(Q,n)=\begin{cases}\frac{\tilde{D}(O+C+nF)}{nQ}+\tilde{D}V+\frac{P_d}{2n}(O+C+nF)\\+\frac{Q}{2}\left\{(H_b+C_bP_d+VP_d)+\left[\left(\frac{(2-n)\tilde{D}}{P}+n-1\right)(H_s+C_sP_d)\right]\right\}\end{cases}\text{ ------(4)}$$

Our aim is to apply graded mean method to defuzzify the fuzzy total cost, and then obtain the optimal order quantity Q^* by using simple calculus technique. Suppose D = (D_1, D_2, D_3, D_4), is a fuzzy trapezoidal numbers,

$$I\tilde{T}C(Q,n)=\begin{cases}\frac{(D_1+2D_2+2D_3+D_4)(O+C+nF)}{6nQ}+\frac{(D_1+2D_2+2D_3+D_4)}{6}V+\frac{P_d}{2n}(O+C+nF)\\+\frac{Q}{2}\left\{(H_b+C_bP_d+VP_d)+\left[\left(\frac{(2-n)(D_1+2D_2+2D_3+D_4)}{6P}+n-1\right)(H_s+C_sP_d)\right]\right\}\end{cases}$$

--(5)

$$\frac{\partial ITC(Q,n)}{\partial Q}=0$$

We get,

$$Q^*=\sqrt{\frac{(D_1+2D_2+2D_3+D_4)(O+C+nF)}{3n\left\{(H_b+C_bP_d+VP_d)+\left[\left(\frac{(2-n)(D_1+2D_2+2D_3+D_4)}{6P}+n-1\right)(H_s+C_sP_d)\right]\right\}}}$$

Algorithm for Finding Fuzzy Total Cost and Fuzzy Optimal Order Quantity:

Step 1: Calculate total cost I*TC(Q, n)* for the crisp model as given in eq.(1) for the given crisp values .

Step 2: Now, determine fuzzy total cost $I\tilde{T}C(Q,n)$ using fuzzy arithmetic operations on fuzzy Demand , taken as fuzzy trapezoidal numbers.

Step 3: Use graded mean method for defuzzification of $I\tilde{T}C(Q,n)$. Then, find fuzzy optimal order quantity Q^*, which can be obtained by putting the first derivative of $ITC(Q,n)$ equal to zero and where second derivative of $ITC(Q,n)$ is positive at $Q = Q^*$.

4. Numerical Example

Example:

Crisp Model:

Following parameters are to be taken in appropriate units:

P=10000units/ year , C=800/batch , H_b = $7 /unit / year , H_s = $6/unit / year D=4800 units/ year , O =$25/order , F =$50 /delivery,V=$1/unit ,$C_b$=50/unit, P_d =164.547,n=6 / production batch cycle. The total cost ITC(Q,n)= **15757.9**, Q= **164.547**

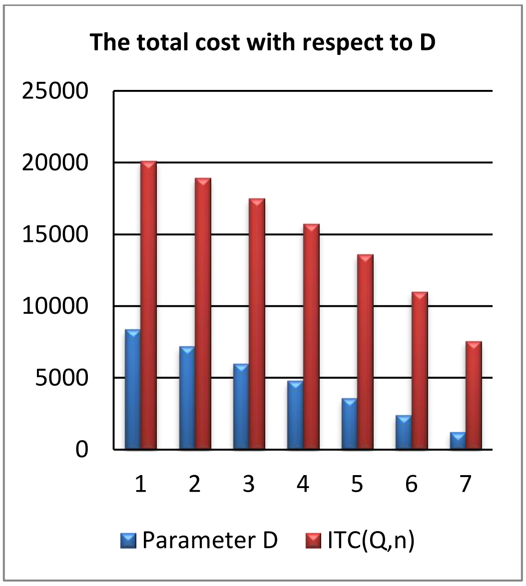
Fuzzy Model:

Let demand per unit time be a trapezoidal number. D=(4500,4600,4900,5300)

P=10000units/ year , C=800/batch , H_b = $7 /unit / year , H_s = $6/unit / year , O =$25/order , F =$50 /delivery,V=$1/unit ,C_b=50/unit, P_d =164.547,n=6 / production batch cycle. The total cost ITC(Q,n)= **15757.9**, Q^* = **164.547**

S.NO	Change of parameter in %	Parameter D	Q	ITC(Q,n)	Parameter n	Q	ITC(Q,n)
1	75	8400	269.284	20116.4	11	108.983	16138.4
2	50	7200	229.819	18967.1	9	121.956	15965.8
3	25	6000	195.615	17520.9	8	139.474	15828.8
4	0	4800	**164.547**	**15757.9**	**6**	**164.547**	**15757.9**
5	-25	3600	134.922	13624.5	5	203.67	15821.5
6	-50	2400	104.867	11001.1	3	274.125	16214.2
7	-75	1200	70.9	7565.72	2	445.362	17793.3

Table 1, **The total cost and order quantity with respect to D and n**



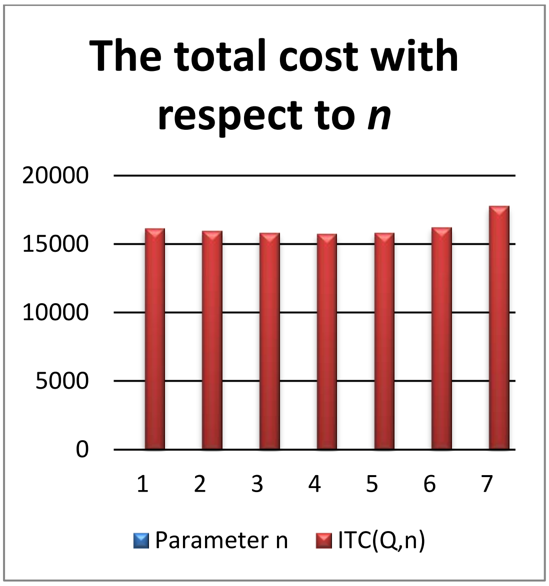


Fig.1.**The total cost with respect to *D***

Fig.2.**The total cost with respect to n**

5.Conclusion

This document presents a fuzzy inventory EPQ model to be calculated during the production and storage of a product. The proposed model is developed in both fuzzy and crisp environments. In the fuzzy environment, the demand of the inventory parameters is assumed to be trapezoidal fuzzy numbers. For classified defuzzification, the graded method is used to evaluate the optimal total cost and the optimal quantity. From a digital example, this fuzzy and crispy pattern was tested.

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