

# SINGLE INVENTORY MODEL UNDER FUZZY ENVIRONMENT WITH DEMAND AND SHORTAGE LEVEL

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## Abstract

In this paper, the inventory model was considered to be deficient in fuzzy environments using new pentagonal fuzzy numbers. Our goal is to determine fuzzy production volume and fuzzy minimum total cost by using Pascal triangle graded mean defuzzification for the proposed inventory model. Shortage level, production cost and total demand quantity are taken based on pentagonal fuzzy numbers. A relevant numerical analysis is also included, to justify the notion.

**Keywords :** *Pentagonal Fuzzy Numbers, Fuzzy Production Quantity, Fuzzy Minimum Total cost, Pascal triangle graded mean.*

## 1.Introduction

In 1915, Harris developed the first inventory model [3]. Then in 1965, for the first time comment. Zadeh introduced fuzzy sets [11]. Theory collections have attracted the attention of many researchers. In 1983, Urgeletti Tinarelli [9] proposed inventory control models and issues. In 1987, Park [5] proposed Model of the theoretical explanation of the vague set of economic order quantity inventory problem. In 2002, Hsieh [4] proposed an approach to improving fuzzy production inventory models. In 2012, Dutta and Pawan Kumar [2] delivered a fuzzy inventory Defect-free model using trapezoidal fuzzy number with sensitivity analysis. In 2000, Der-Chen Lynn and Jing-Shing Yao [1] proposed fuzzy economic production for production inventory models. In 2015, Stephen and Rajesh [7] were formed Fuzzy inventory model with allowable deficit using hexagonal fuzzy

numbers. In this article, In Section 2, we briefly describe the concepts and assumptions used in developed fuzzy Inventory model. In Section 3, the proposed inventory model in crisp and fuzzy sense. In section 4, algorithm presented in Crisp and fuzzy Sense. In Section 5, a numerical example is given to justify the proposed ideas. In Section 6, The Conclusions are also included.

## 2. Mathematical model

### 2.1 Notations

d =demand per unit item

h=holding cost per unit item

s=shortage level

L=lot size

c=set-up cost per cycle

v=shortage cost per item

### 2.2 Assumptions

- 1) Replenishment is instantaneous
- 2) Lead time is zero
- 3) Unit price is related to the demand as  $p = \gamma e^{-kd} d$  where  $\gamma > 0, k > 0$  are real constants.

## 3. Proposed inventory model under crisp and fuzzy sense

Let us consider the proposed inventory crisp model total cost is,

$$TIC(L,s) = \gamma e^{-kd} d + \frac{h(L-s)^2}{2L} + \frac{vs^2}{2L} + \frac{cd}{L}$$

For the crisp order quantity

$$L = \sqrt{\frac{2cd + s^2(h+v)}{h}}$$

Here d,h and v are taken as fuzzy environments.

We suppose  $d = (d_1, d_2, d_3, d_4, d_5), h = (h_1, h_2, h_3, h_4, h_5), v = (v_1, v_2, v_3, v_4, v_5)$  are non negative pentagonal fuzzy numbers. Our goal is to obtain fuzzy total and order quantity in terms of pentagonal fuzzy numbers.

Then the inventory crisp model total cost becomes,

$$TIC(L) = \gamma e^{-k\tilde{d}} \tilde{d} + \frac{\tilde{h}(L-s)^2}{2L} + \frac{\tilde{v}s^2}{2L} + \frac{c\tilde{d}}{L}$$

By applying arithmetic function and simplifying we get,

$$TIC(L) = \begin{bmatrix} \gamma e^{-kd_1} d_1 - h_1 s + \frac{h_1 L}{2} + \frac{1}{2L} (2cd_1 + s^2 (h_1 + v_1)), \\ \gamma e^{-kd_2} d_2 - h_2 s + \frac{h_2 L}{2} + \frac{1}{2L} (2cd_2 + s^2 (h_2 + v_2)), \\ \gamma e^{-kd_3} d_3 - h_3 s + \frac{h_3 L}{2} + \frac{1}{2L} (2cd_3 + s^2 (h_3 + v_3)), \\ \gamma e^{-kd_4} d_4 - h_4 s + \frac{h_4 L}{2} + \frac{1}{2L} (2cd_4 + s^2 (h_4 + v_4)), \\ \gamma e^{-kd_5} d_5 - h_5 s + \frac{h_5 L}{2} + \frac{1}{2L} (2cd_5 + s^2 (h_5 + v_5)) \end{bmatrix}$$

$$= F(\tilde{L}).$$

**Pascal triangle graded mean defuzzification for pentagonal fuzzy numbers**

Let  $A = (a_1, a_2, a_3, a_4, a_5)$  be pentagonal fuzzy numbers then we can take the coefficient of fuzzy numbers from pascal’s triangles and apply the basic likelihood approach then we get the accompanying formula  $P(A) = \frac{a_1 + 4a_2 + 6a_3 + 4a_4 + a_5}{16}$ . The coefficients of  $a_1; a_2; a_3; a_4; a_5$  are 1, 4, 6, 4, 1.

Now ,we defuzzifying the total cost using Pascal triangle graded mean method, we get

$$TIC\tilde{(L)} = \frac{1}{16} \begin{bmatrix} \gamma e^{-kd_1} d_1 - h_1 s + \frac{h_1 L}{2} + \frac{1}{2L} (2cd_1 + s^2 (h_1 + v_1)) + \\ \gamma e^{-k4d_2} 4d_2 - 4h_2 s + \frac{4h_2 L}{2} + \frac{1}{2L} (2c4d_2 + s^2 (4h_2 + 4v_2)) + \\ \gamma e^{-k6d_3} 6d_3 - 6h_3 s + \frac{6h_3 L}{2} + \frac{1}{2L} (2c6d_3 + s^2 (6h_3 + 6v_3)) + \\ \gamma e^{-k4d_4} 4d_4 - 4h_4 s + \frac{4h_4 L}{2} + \frac{1}{2L} (2c4d_4 + s^2 (4h_4 + 4v_4)) + \\ \gamma e^{-kd_5} d_5 - h_5 s + \frac{h_5 L}{2} + \frac{1}{2L} (2cd_5 + s^2 (h_5 + v_5)) \end{bmatrix}$$

Differentiating partially w.r.t L and equating it to zero.

$$\frac{\partial TIC\tilde{(L)}}{\partial L} = 0.$$

Therefore ,

$$\tilde{L}^* = \sqrt{\frac{2c(d_1 + 4d_2 + 6d_3 + 4d_4 + d_5) + s^2 [(h_1 + 4h_2 + 6h_3 + 4h_4 + h_5) + (v_1 + 4v_2 + 6v_3 + 4v_4 + v_5)]}{(h_1 + 4h_2 + 6h_3 + 4h_4 + h_5)}}$$

Also  $\tilde{L} = \tilde{L}^*$  we have  $\frac{\partial^2 TIC\tilde{(L)}}{\partial L^2} > 0$ ; this show that  $TIC\tilde{(L)}$  is minimum at  $\tilde{L} = \tilde{L}^*$

#### 4.Algorithm

Algorithm for detecting fuzzy total cost and fuzzy optimal quantity.

Step 1: Calculate the sample fuzzy total cost for fuzzy values of d,h and v.

Step 2: Now New Arithmetic Functions Determine the fuzzy total cost using fuzzy demand, shortage level cost and fuzzy shortage Cost taken in terms of pentagonal fuzzy numbers.

Step 3 : Find the fuzzy optimal order quantity which can be obtain by putting the first derivative of TIC(L) equal to zeroand second derivative is positive at  $\tilde{L} = \tilde{L}^*$

## 5. Numerical example

### 5.1 Crisp model

Let  $\gamma = 2$ ,  $k = 0.1$ ,  $d = 500$  /unit,  $s = 3$ ,  $c = 80$ ,  $v = 10$  /unit,  $h = 0.7$ /unit.

The solution of crisp model is

$L = 338.2651$ ,  $TIC(L) = 234.6855$ .

### 5.2 Fuzzy model

Let  $d = (300, 400, 500, 600, 700)$ ,  $h = (0.5, 0.6, 0.7, 0.8, 0.9)$ ,  $v = (8, 9, 10, 11, 12)$

The solution of fuzzy mode can be determined by Pascal triangle graded mean method.

$L = 338.2651$ ,  $TIC(L) = 234.6855$ .

### 5.3 Sensitivity Analysis

A sensitivity analysis is performed to study the effects of change in parameters  $d$  and  $v$ .

**Table 1.  $d$  vs total cost and  $v$  vs total cost**

$d$	TIC(L)	$v$	TIC(L)
600	257.2596	11	234.6989
700	278.0203	12	234.7122
800	297.3452	13	234.7255
900	315.4963	14	234.7388
1000	332.6647	15	234.7521

From the above observation we concluded as follows:

- \* Total cost increases as the demand rate increases.
- \* Total cost increases as the shortage cost increases.
- \* Total cost obtained by Pascal triangle graded mean method is equal to the crisp total cost.

## 6. Conclusion

In this paper, we read about fuzzy economic production in the commodity model with the help of pentagonal fuzzy numbers. Various fuzzy optimal sizes, demand, shortage cost and shortage level cost pentagonal fuzzy numbers are estimated. To find the fuzzy total cost Pascal triangle graded mean method has been used. A sensitivity analysis is also conducted to know the behavior of changes in parameters.

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